

## Mathematical Understanding Scale: High School Level Mathematics

**Note:** The descriptors for each level identify many of the key indicators that a student is developing mathematical understanding. Judgments about student placement on a level are to be based on overall attainment, not on a point-by-point check off.

				
<p><b>Level 9</b></p> <p>Understands and uses properties (e.g., closure, associative, distributive) and inverse relationships (e.g., multiplication/division, square/square roots) to solve problems. Solve problems involving fractions, decimals, and percents. Developing understanding of and uses proportional reasoning to solve a variety; of problems (e.g., rates, similarity, probability, scaling). Developing an understanding of large numbers, and their various representations (e.g., exponential, scientific, calculator notation).</p> <p>Represents functional relationships expressed in contexts, models, and/or symbolically in several ways and explains how different situations or models may represent the same function.</p>	<p><b>Level 10</b></p> <p>Understands and uses concepts and properties of the real number system (e.g., factorization, irrational numbers, density, additive and multiplicative inverses). Analyzes and compares algorithms for computing with rational numbers (including integers) and uses efficient and effective methods for solving problems. Uses proportional reasoning to solve a variety; of problems including situations involving direct/inverse proportions and right triangle trigonometry.</p> <p>Developing understanding of and ability to represent/solve problems using algebraic expressions/equations and their graphs. Flexibly moves between numerical, algebraic, and geometric representations of a problem situation.</p>	<p><b>Level 11</b></p> <p>Solves problems with irrational numbers. Makes generalizations about the real number system and its properties to explain its structure. Justifies the process and solutions to problems involving basic number theory concepts (e.g., why powers of 2 cannot be written as the sum of consecutive integers).</p> <p>Developing understanding of and facility with algebraic computations. Flexibly moves between numerical, algebraic, and geometric representations and solutions of a problem.</p>	<p><b>Level 12</b></p> <p>Beginning to understand the concept of complex numbers as they arise as solutions to polynomial equations. Represents quantitative relationships using equations and their graphs (e.g., linear, quadratic, exponential, periodic), and solves related problems. Understand and solves problems involving systems of equations and interprets the solutions in the context of the situation and/or graphically.</p>	<p><b>Level 13</b></p> <p>Understands and explains how the real and complex number systems are related (e.g., properties, structure, representations). Understands and explains the relationships between the real number system and the mathematical structure of algebra (e.g., operations, properties, axioms). Fluently uses algebraic expressions, equations, functions, and graphs to solve complex problems. Generalizes beyond the original situation to other situations. Uses the functional relationships to examine in more detail specific characteristics of the function (families of functions).</p>

## High School Level Mathematics (Continued)

<b>Level 9</b>	<b>Level 10</b>	<b>Level 11</b>	<b>Level 12</b>	<b>Level 13</b>
<p>Knows and/or derives appropriate formulas for area, perimeter, volume, and surface area and uses them, as well as mental visualizations, to solve problems. Uses a variety of tools to accurately measure and construct geometric figures.</p>	<p>Uses visualizations and/or other representations of geometric figures (e.g., geometric decompositions and transformations), and appropriate formulas to analyze, interpret and solve problems. Beginning to understand the difference between multiple observations of phenomena and a sound mathematical argument. Developing understanding of the approximate nature of measurement and the degree of accuracy needed.</p>	<p>Identifies necessary and sufficient conditions for determining/defining a geometric figure, elements of a figure, or relationships among figures, and uses inductive reasoning to justify conjectures about geometric figures. Understands proofs of the Pythagorean Theorem and uses the theorem and right triangle trigonometric relationships to analyze and solve problems involving geometric figures.</p>	<p>Explains why a particular representation or interpretation is most appropriate for a given problem situation; and makes simple deductive mathematical arguments to support conclusions or to explain relationships between geometric figures. Understands and explains the difference between deductive proof and inductive reasoning.</p>	<p>Makes logical mathematical arguments and writes sophisticated mathematical proofs.</p>
<p>Understands and uses simple tools of statistical analysis (e.g., range, measures of central tendency, box and whisker graphs) to interpret single variable data sets. Beginning to represent and interpret two variable data sets using scatter plots. Recognizes and explains the differences between theoretical probabilities and the results of experimental trials.</p>	<p>Analyzes and critiques data reports and explains how they might have been affected by considering possible sources of bias in the design, data collection and/or representation. Uses theoretical probability to predict and explain the outcomes of problems involving probability.</p>	<p>Beginning to develop understanding of statistical measures of variation including standard deviation. Beginning to understand, represent, and interpret two variable data sets. Analyzes and critiques data reports (e.g., reliability of sampling procedures, bias, missing or incorrect information). Developing understanding and finds probabilities of compound and conditional events.</p>	<p>Understands and uses descriptive statistics to analyze and interpret more complex situations. Predicts trends for two variable data sets using lines and curves of best fit and their equations. Developing understanding of and finds theoretical probability for complex situations.</p>	<p>Developing understanding of and uses inferential statistical concepts (e.g., normal and chi square distributions). Designs and conducts a statistical study comparing two populations, including formulating and testing of hypotheses. Understands, uses, and interprets correlation coefficients for lines of best fit.* Flexibly uses theoretical probability to solve problems.</p> <p>*Some students may not have had the opportunity to learn the statistics content. If students meet the other criteria, level 13 is an appropriate placement.</p>