THE CENTER FOR
Language in Learning

Chapter 2
The Learning Record Mathematics Scales

The Learning Record scales describe what students should understand about mathematics and be able to do as they progress from year to year. There are two mathematics scales—a Mathematical Understanding Scale (grades K through high school) and a Mathematical Disposition Scale (grades 3 through high school). Both scales are aligned with mathematics standards developed by the National Council of Teachers of Mathematics (NCTM): *Principles and Standards for School Mathematics* (PSSM, 2000) and the earlier version, *Curriculum and Evaluation Standards* (1989). Therefore, they should be congruent with most state and local standards.

While one purpose of the LR mathematics scales is to determine student placement at the end of the year, they also are useful during the year. The Mathematical Understanding Scale descriptors help students, parents, and teachers know the content expectations to be reached by the end of the year. The Mathematical Disposition Scale descriptors help students, parents, and teachers know the learning behaviors and characteristics are associated with high achievements. Both scales help identify what students might work on to improve their learning. The scales also serve as a guide for teachers' long-term planning. The scales are included at the end of the chapter.

Assumptions Underlying the Mathematics Scales
There were a number of assumptions underlying the development of the mathematics scales.

- <u>Mathematical understanding is critical</u>. Developing student understanding of mathematics was the foundation for the development of the scales—

students need to understand and make sense of the mathematics they learn. It is not sufficient for students to be able to do procedures by rote. Students must know when and how to use mathematics, and they must think critically when solving problems. Problem solving is embedded throughout the scales at all levels.

- <u>Expectations are high, but realistic.</u> The scales were written with the assumption that ALL students can learn mathematics provided they participate in a quality mathematics program with teachers who understand mathematics and teach in ways that help students develop understanding and competence. As stated earlier, the NCTM Standards were used as references in the development of the scales. The developers of the scales also drew upon their many years of working with students to describe levels they believe are high, yet realistic and attainable.

- <u>Important mathematical ideas must be apparent</u>. On the Mathematical Understanding Scale, mathematics content is described in these four areas or strands: (1) number sense; (2) patterns, functions, and algebra; (3) geometry and measurement; and (4) data analysis, statistics, and probability. At each level of the scale, emphasis was placed on describing the important mathematical ideas for that level. However, since the scales descriptors are brief and since there is generally just one level per grade, not all the important mathematical learnings for each level can be specified. Students will need to have many classroom experiences with content not specifically mentioned at a particular scale level. In the scales, emphasis was placed on describing the *key* indicators that reveal that a student is developing mathematical understanding.

- <u>The scales are developmentally-based</u>. The scales were designed to describe levels that are distinct enough from one another to show progress in learning, with approximately one level on the Understanding Scale per grade level. However, students will progress on the scales at different rates. Since student learning builds on what students already know, understand, and can use, and building and deepening understanding of im-

portant mathematical ideas requires many experiences over a long period of time, students will be working at a level for an extended period of time. While not all students may progress a full level during a school year, the scales do provide a way for showing student growth from the beginning to the end of the year and across school years.

- Students' disposition toward mathematics must be reflected. As mentioned in Chapter 1 (pages 3-4), *Five Dimensions of Learning* are integral to the Learning Record: Confidence and Independence, Experience, Skills and Strategies, Knowledge and Understanding, and Ability to Reflect. These dimensions interact together in complex ways to promote student learning. The Mathematical Disposition Scale was developed to incorporate all five dimensions of learning. Although there will be some relationship between students' level placement on the two mathematics scales, two students who are at the same level on the Understanding Scale may be at quite different levels on the Disposition Scale. For example, a high-achieving fifth grade student and a low-achieving seventh grade student may be working at the same level of the Mathematical Understanding Scale; however, it is probable that they differ significantly in their mathematical disposition. The fifth grader may be confident and work independently, while the seventh grader, who is working below grade level expectations, may lack confidence and must be urged to complete his or her work.

- Holistic judgments will be used for student placement. Although each level of the scales describes a number of different learnings in mathematics, an overall judgment, based on multiple pieces of evidence in a student's portfolio, is used to place a student at a level. The description of each level is not intended to be a check-list with the requirement that the student must have mastered each criterion mentioned. Nor is it realistic to expect that a portfolio can supply evidence of every mathematical idea that is mentioned in a level descriptor. Generally, though, what is included

in the portfolio, student work and teacher/student commentary, should be supportive of placement at a specific level.

The Mathematical Understanding Scale

The Mathematical Understanding Scale (for grades K through high school) identifies the important mathematics that students should understand and be able to do in the areas of: (1) number sense; (2) patterns, functions, and algebra; (3) geometry and measurement; and (4) data analysis, statistics, and probability.[*] Problem solving is integrated throughout all of these mathematical areas. The scale is divided into thirteen levels, grouped into five levels per grade span, K-2, 3-5, 6-8, and high school level courses. For ease of use, the levels of the grade spans overlap as follows.

Grades K-2: Levels 1 to 5

Grades 3- 5: Levels 4 to 8

Grades 6-8: Levels 6 to 10

High School: Levels 9 to 13

While there naturally will be variation in attainment among different students in a class, the grade level expectations of the mathematical understanding scale are as follows.

- *Grades K-8.* Given a good mathematics program, the end-of-year expectation is for students in grades 1-8 to be performing two levels higher on the Mathematical Understanding Scale than their grade placement. For example, the expectation is that 4th grade students who meet grade level standards will be at Level 6 of the Learning Record Mathematical Understanding Scale. The expectation for kindergarten students is level 2.

- *High School Level Courses.* The last grouping of the scale is for students enrolled in courses that have been considered high school level "college prep" (i.e., courses that go beyond traditional middle school level math). Some students may begin these courses in grades 8 (or lower) and other students may wait till later. While it is recognized that the title, structure, and content of courses will vary across schools (Algebra 1, Integrated

Math 1, etc.), the descriptor for each level includes content from all mathematics strands. (The areas of number sense and patterns, functions, and algebra have been combined in levels 12 and 13.) By the end of three years of challenging high school level mathematics courses, the expectation is for students to be performing at level 12. (The content of Advanced Placement (AP) courses is not included in the scale, although the expectation is that students enrolled in these courses would be at performing at level 12 or 13 on the scale.)

The Mathematical Disposition Scale

The Mathematical Disposition Scale (for grades 3 to high school) describes other important characteristics necessary for students to be successful at learning mathematics. It incorporates the LR Five Dimensions of Learning and is used in conjunction with the Mathematical Understanding Scale to indicate student progress. The scale also reflects the NCTM Process Standards for Problem Solving,[*] Communication, Connections, and Representation.

To some extent, the level descriptors on the Mathematical Disposition Scale are developmental in nature. For example, the ability to analyze a problem situation or critically reflect on the elegance of a solution will develop as a student matures and successfully encounters and solves more complex mathematical problems. For this reason, almost no third or fourth grade high achieving students will be working at the highest level, level 5.

Since there are only five levels on the Mathematical Disposition Scale, which are used for all grades 3-12, there is no expectation that students will advance levels at a particular rate. A student could stay at the same level for several years. However there is an assumption that there will be some relationship between students' placement on the Mathematical Understanding Scale and the Mathematical Disposition Scale. For example, it is almost impossible to imagine how a student whose placement on the Mathematical Understanding Scale is two

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[*] The NCTM Content Standards are identified as: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

or three levels below grade level expectations could be functioning at level 4 or 5 on the Mathematical Disposition Scale.

However, students who seem to be at the same level of mathematical knowledge will often vary in their degree of confidence, willingness to use and understand different approaches, and their ability to reflect on their work. For example, two fifth grade students may be at level 7 on the understanding scale, but one may be at a level 2 on the disposition scale and the other at level 4. A teacher should consider these differences when making instructional decisions. For example, a student who consistently uses only one strategy to solve problems may be encouraged to work with another student who uses a different approach and/or be asked to restate or explain strategies used by others.

To Do*: Becoming Familiar with the Mathematics Scales
It is important for all who will be involved with the Learning Record—teachers, students, and parents—to become familiar with the two mathematics scales. While students will not be placed at a level of the scales until near the end of school year, knowing what the scales include can be helpful for teachers in planning their program, deciding what to include in student portfolios, helping students to know what is important for the year in terms of mathematical understanding and disposition, and conferring with and informing parents of their child's progress. With use over a period of years, teachers will internalize the essence of the descriptors for their grade level.   To become familiar with the scales, you may want to do the following.

- *Become knowledgeable about the Principles and Standards of School Mathematics* (NCTM, 2000). The PSSM should be used as a reference while becoming familiar with the scales. It contains good explanations of the NCTM's principles and standards overall and by grade span: Pre-K–2, 3–5, 6–8, and 9–12. It also provides many examples of problems and how teachers might help students learn and understand many mathematical ideas.

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[*] Problem Solving is also integrated throughout the LR Mathematical Understanding Scale.

- *Read and discuss with other teachers what is meant by the LR scales de-scriptors.* Focus on the LR scale levels for your grade, but also pay attention to how mathematical ideas progress from level to level. (Since only the key ideas for a grade are included in a level descriptor, not all concepts cannot be tracked across all levels. For example, even though it is expected that students will have some experiences with fractions in grades K-2, they are not specifically mentioned in the descriptors for Levels 1-4.) Use the PSSM and other references to discuss and help clarify the intended meaning of the LR descriptors. Expect that understanding and internalizing the scale descriptors will be a long term process.

- *Compare the level descriptors on the scales with your local and state standards.* Because the scales are designed to be quite brief, the standards your school or district is using are likely to be much more detailed than the scale descriptors. In general, the sentences that make up the scale descriptors are a shorthand for complex understandings that students performing at a level should exhibit and do not specify the experiences students would need to develop these understandings. Since standards vary some from district to district and from state to state in both their interpretation and their substance, the LR scales allow for adaptation in use at the local level while maintaining comparability across district and state lines.

- *Discuss with other teachers what kind of student work would provide evidence of competency, what kinds of classroom experiences would promote student competence.* Examine student exemplars from the Center for Language in Learning or you may want to bring in samples of student work from your classrooms and compare them to examples from the PSSM and this handbook. Further discussions can focus on what inferences can by made by looking at the work and the student/teacher commentary about the work.

  - *Compare the scales with the curriculum and assessment materials you are using.* What units or experiences in your curriculum have the po-

tential for providing evidence of student progress that are particularly related to the scale descriptors?

- *Share the scales with students and their parents.* Use the Mathematical Understanding and the Mathematical Disposition Scales to communicate your expectations to students and their parents. Share your expectations for students' mathematical understanding at the end of a school year and, by using the Disposition Scale, share the kinds of behaviors associated with high achievement in mathematics. You may want to point out how the Disposition Scale relates to the NCTM Standards for Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.